Growth of Business Services: A Supply-Side Hypothesis

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Abstract

The paper aims to explain why/how the services sector may grow faster than manufacturing. It develops a two-sector, closed-economy model, having a manufacturing sector and a services sector. Accumulation of human capital serves as the basis of growth. The analysis focuses on business services, while household services are also considered. It is argued that differences in returns to scale between the two sectors and employment frictions in manufacturing explain why the growth rate of the services sector may be higher. The model also features that within the services sector the business services sub-sector may grow faster than household services.

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1 Introduction

It is a well known that, in terms of both output and employment, the services sector has overtaken manufacturing as the leading sector in many modern economies.[1] In the growth literature this phenomenon has been attributed to uneven growth in total factor productivity in market and home production of manufacturing and services goods (Ngai and Pissarides (2008)), non-homothetic preferences and rise in income (Eichengreen and Gupta (2009)), and, growing demand for skill intensive services with income (Buera and Kaboski (2012)). Less known are patterns of growth within the service sector. Various services can be categorized into three types: pure business services, pure consumer services and the 'hybrid' (consumed by both firms and households). Consumption or business services in total then consist of pure consumption or business services and some of the hybrid.

Growth rates of these sub-sectors are hardly uniform. In U.S., U.K. and Japan for instance, the share of pure business services in the services sector as a whole has nearly or more than doubled in a span of over three decades 1970-2006. In 2006 pure business services formed 15% to 20% of the total services - a small but a significant proportion (according to EU KLEMS data).

Table 1: CAGR (in %) of Sectoral Outputs and Employment (1970-2006)

	Output Growth			*Employment Growth*		
	US	UK	Japan	US	UK	Japan
Utilities	1.1	3.1	3.2	-0.4	-2.3	- 0.5
Construction	1.6	1.7	0.3	2.1	-0.3	0.7
Wholesale and Retail Trade	3.8	3.0	3.1	1.5	0.8	1.5
Hotels and Restaurants	3.0	2.0	2.3	2.7	1.8	2.7
Transport and Communication	3.6	3.9	3.0	0.8	-0.2	0.5
Pure Business Services	**5.9**	**6.0**	**5.1**	**4.9**	**3.1**	**5.1**
Pure Consumer Services	**2.8**	**2.1**	**3.1**	**1.9**	**1.4**	**2.3**
Manufacturing	**1.9**	**0.1**	**2.2**	**-1.0**	**-2.7**	**-0.4**

Source: EU KLEMS

Table 1 records the compounded annual growth rates (CAGR) of the sub-sectoral real (gross) output as well as employment within the service sector vis-a-vis manufacturing in these countries over the period 1970-2006.[2,3] Observe that *in terms of output and employment, pure business services have grown faster than pure consumer services, which in turn have grown faster than manufacturing.*

We treat this as a stylized fact, and, the objective of this paper is to focus on business services and provide a rationale behind the above stylized fact.

A number of studies have attributed the rising share of services in GDP to preference changes accompanying economic development. In the long run, the argument goes, the rise in real income shifts demand from agricultural goods to manufacturing goods and then to services.[4] The manufacturing sector outgrowing the agricultural sector is understandable in terms of the preference-shift hypothesis. But the services sector – especially the business service sub-sector – outpacing manufacturing is not explained by this hypothesis, since the argument is applicable to consumer services. How a *derived* sector like business services may grow faster than its *parent* sector, manufacturing, is not too obvious. It is also not apparent how the growth rate of business services may exceed that of consumer services. This paper develops a theoretical model which accords with the above stylized fact via two supply-side assumptions or mechanisms.
A: *Returns to scale are less in manufacturing compared to the services sector.*
B: *Adjustment of employment is more sluggish in manufacturing than in the services sector.*

In view of A, it is easy to see how the latter may grow faster than the former. Suppose all services are business services, produced by one input, labor, and, output in the services sector is related one-to-one with labor employment in that sector (CRS technology). In contrast, let manufacturing output be a function of labor and (business) services under a decreasing-returns technology. Suppose that in the steady state employment grows at the same rate between the two sectors. It follows immediately that employment and output in the service sector grow at the same rate, while manufacturing output grows at a lesser rate. That is, the business services sector, whose existence is derived from demand by manufacturing, can grow faster than manufacturing. The same argument goes through even if both sectors may be subject to decreasing returns to scale as long as the scale elasticity is lesser in manufacturing.

Suppose, in addition, there are labor or worker frictions, and, they are more prevalent in manufacturing than in the services sector, implying that adjustment of employment is more sluggish in the manufacturing sector. In a growth scenario it would then imply that employment in the services sector would grow faster than that in manufacturing.

Our analysis indeed yields something more subtle, that is, assumptions A and B 'deliver' that business services would grow faster than consumer services, which, in turn, would grow faster than manufacturing. Intuitively, Assumption A (difference in returns to scale) implies, per se, that business and consumer-service outputs would grow at the same rate, which is higher than that of manufacturing. Assumption B tends to imply a higher growth rate of employment in the business-services sector than in manufacturing, which constitutes an additional source of higher output growth rate of business services – but not for consumer services – compared to manufacturing. As a result, the growth rate of consumer services in terms of employment and output falls short of that business services but exceeds that of manufacturing.[5]

Assumptions A and B, both, are empirically motivated. There are numerous empirical studies on returns to scale in various industries, yielding different results owing to differences in data and methodology; for an overview, see (WDR, 2009, Chapter 4). In particular, Basu et al. (2006) is one of the few which estimate returns to scale for industries in both manufacturing and services. This U.S. economy based study finds that for manufacturing as a whole, there is evidence of decreasing returns in terms of gross output, less so for value-added. Within manufacturing, durable

Table 2: Estimates of Returns to Scale of Selected Industries

Durable Manufacturing		*Nondurable Manufacturing*		*Nonmanufacturing*	
Lumber (24)	0.51	Food (20)	0.84	Construction (15-17)	1.00
Furniture (25)	0.92	Tobacco (21)	0.90	Transportation (40-47)	1.19
Stone, clay, & glass (32)	1.08	Textiles (22)	0.64	Communication (48)	1.32
Primary metal (33)	0.96	Apparel (23)	0.70	Electric utilities (491)	1.82
Fabricated metal (34)	1.16	Paper (26)	1.02	Gas utilities (492)	0.94
Nonelectrical machinery (35)	1.16	Printing & publishing (27)	0.87	Trade (50-59)	1.01
Electrical machinery (36)	1.11	Chemicals (28)	1.83	FIRE (60-66)	0.65
Motor vehicles (371)	1.07	Petroleum products (29)	0.91	Services (70-89)	1.32
Other transport (372-79)	1.01	Rubber & plastics (30)	0.91		
Instruments (38)	0.95	Leather (31)	0.11		
Miscellaneous manufacturing (39)	1.17				
Column Average	**1.01**		**0.87**		**1.16**
Median	**1.07**		**0.89**		**1.10**

Source: Basu et al. (2006); reproduced here with permission.
FIRE stands for finance, insurance, and real estate.

manufacturing exhibits increasing returns to scale while there are decreasing returns in non-durable manufacturing. Scale elasticities of services production exceed unity and are higher than those in durable manufacturing. For reference, Table 2 reproduces Table 1 in Basu et al. (2006), in which the last column (nonmanufacturing) contains various service industries.[6] Notice that service industries like transportation, communication and personal services have higher returns to scale than all manufacturing industries, except chemicals.

For Japan, Morikawa (2011) finds evidence of increasing returns to scale for ten major personal services industries. He attributes this to knowledge spillover effects due to localization or agglomeration. It is also easy to 'see' strong scale economies in services like transportation, communication and utilities, having substantial overhead costs and relatively low marginal costs. There are several other services-sector-specific studies on measurement of returns to scale. Scale economies are also found for retail trade in Israel (Ofer (1973)), banking and finance in the U.S. (McAllister and McManus (1993)) and hospital industry in the U.S. (Berry (1967) and Wilson and Carey (2004)).

Apart from agglomeration or technology factors, a highly plausible underlying factor behind returns to scale in services being higher compared to manufacturing may be the scarcity of land and differences in the intensity of land use in production. In recent decades land has become a major issue in the expansion of manufacturing. Acquiring land has become increasingly costly and growing environmental regulations have led to stringent limitations for the use of acquired land towards industrial activities. In the context of growth of manufacturing in China and India, Srivastava (2007) and *Business Line* (2012) express that availability of land is one the reasons why manufacturing sector in China has grown much faster than in India. But, land is not so much of a constraint for service production units. For example, in a study of 15 major countries of the European Union, Hubacek and Giljum (2003) find that 2.1 million hectares of productive land is under manufacturing while only 1.1 million hectare is used for the services sector. Differential land constraints would imply differential returns to scale.

Turning to worker frictions, they seem to vary directly with firm size, via congestion, unionization and employment protection laws (EPLs). As the average firm size is larger in manufacturing than in services, worker frictions would tend to be more prevalent in manufacturing. According to Bureau of Labor Statistics (BLS) 2010, in U.S. about 34% of manufacturing enterprises had more than 20 employees, while the same was true for less than 10% of enterprises in the services sector.[7] OECD database shows that in almost all OECD countries, the average firm size in manufacturing is larger than in services.

Table 3: Business Turnover in Selected Countries

		Enterprise Entry Rate (%)		Enterprise Exit Rate (%)	
Country	Year	Manufacturing	Services	Manufacturing	Services
Italy	2005	5.71	9.47	7.08	8.65
Norway	2005	4.39	7.27	2.96	5.15
Spain	2006	5.77	11.95	6.58	8.61
Canada	2007	4.78	7.9	6.92	8.86
Brazil	2005	9	13	6.73	8.35

Source: OECD database

In 2012, unionization rate (the percentage of employed workers in a sector that had a union or an employee association affiliation) in the U.S. private sector was about 11% in manufacturing and just over 6% in the services sector (BLS). According to ILOSTAT, an ILO database of 165 member countries, in 2010 manufacturing was one of the sectors with the highest number of strikes, while they were the least in the business services sector.

EPLs, which are not sector-specific, stipulate more stringent norms for firms with more than 15 to 20 employees, firms having labor unions or when firms fire workers with long tenure (see Guner et al. (2008) among others). As these conditions prevail more in manufacturing than in services, employment friction is likely to be more in manufacturing.[8] Such cross-sector difference in employment flexibility is also noted by the European Commission in its policy brief European Research Area (2013).

Business turnover rate may be considered as an indirect proxy for labor turnover or employment variability which is inversely related to the degree of worker frictions. Table 3 presents supporting data for five countries – that is, business turnover is less in manufacturing.[9]

Last but not least, it is well-known that labor turnover rates, a direct proxy for employment flexibility, are lower in manufacturing than in the services sectors. To paraphrase Bertola (1992) who analyzed labor turnover costs, "employment is typically quite flexible for small firms and firms in the service sector."[10] Table 4 presents the

Table 4: Labor Turnover Rates for U.S. in 2012

Sector	JOR	JSR
Manufacturing	2.0	1.9
Trade, Transportation and Utilities	2.7	3.3
Education and Health Services	3.2	2.3
Leisure and Hospitality	3.2	5.2
Professional and Business Services	3.1	4.5

JOR: % of workforce recruited on part time or full time basis in a given year;
JSR: % of workforce separated due to quits, layoffs and discharges

Source: Bureau of Labor Statistics.

labor turnover rates in the U.S., in terms of job openings and separation rates (JOR and JSR respectively), for manufacturing and some service industries. JORs and JSRs are higher in service industries.

The rest of the paper proceeds as follows. Related literature is briefly reviewed in Section 2. Our basic model of business services is developed in Section 3. The main result is that output and employment growth rates in the business-service sector exceed those in manufacturing. Consumption services are introduced in Section 4, the central section of the paper. The model therein ranks growth rates of business services, consumption services and manufacturing, and, 'predicts' the stylized fact. Some generalizations and alternative scenarios are explored in Section 5. They include demand shift towards consumption services as income rises via non-homethetic preferences. Section 6 concludes the paper.

2 Related Literature

Starting with Baumol (1967) and Kuznets (1973), the literature on non-balanced sectoral growth that accounts for the relative growth of manufacturing and services has grown.

There are both demand-side and supply-side explanations for non-balanced growth. Kongsamut et al. (2001) and Eichengreen and Gupta (2009) postulate that the observed sectoral output and employment growth trends stem from the differential demand for the goods.

The former develop a growth model with three sectors: agriculture, manufacturing and services. These goods are represented through Stone-Geary preferences where the income elasticity of demand is less than unity for agriculture good, unity for manufacturing good and greater than unity for services. Differential sectoral growth in an economy results from such non-homothetic preferences: the service sector is the fastest growing sector, followed by manufacturing and then agriculture. All sectors share the same production function (implying same factor intensities in equilibrium), except for differences in the (Hicks-neutral) technology parameter. Labor and capital are the factors of production, with the same exogenous labor augmenting technological in all three sectors. Differences in growth rates, however, become narrower over time, and, asymptotically, the growth rates in all sectors are the same. In our model the differences in sectoral growth persist even in long run.

Eichengreen and Gupta (2009) present an empirical study covering sixty countries from 1950 to 2005 that explains the growth of services due to rise in per capita income. They identify two waves of growth of the services sector. The first occurs at per capita incomes lower than USD 1,800 (in year 2000 purchasing power parity dollars). The second starts at per capita income of around USD 4,000 and it stems from the IT revolution and openness of service trade across countries.

Supply side explanations include biased technological progress across sectors (e.g., Ngai and Pissarides (2008)) and differences in production technologies (e.g., Zuleta and Young (2013) and Acemoglu and Guerrieri (2008)).

Ngai and Pissarides (2008), building upon their previous work, namely, Ngai and Pissarides (2007), present a three-sector model with agriculture, manufacturing and services having the same production function, as in Kongsamut et al. (2001), but allowing for differential TFP growth rates: highest in agriculture, next highest in manufacturing and then services. Unlike Kongsamut et al. (2001), preferences are assumed to be symmetric and homothetic. It is the TFP growth ranking which defines the three goods. Higher TFP growth rates in agriculture and manufacturing sectors push capital and labor into the services sector and thus factor shares in the services sector expand at the cost of the other two sectors; see also Krüger (2008).[11] Aggregate ratios like capital to output and consumption to output are, however, constant at the steady state in conformity with Kaldor facts.

In their two-sector model having manufacturing and services, Zuleta and Young (2013) differentiate between the two goods in terms of production function: elasticity of substitution between capital and labor being higher in manufacturing than in services production. In addition, labor-saving technological progress occurs in manufacturing sector. As a consequence, the share of the services sector in employment rises. However, the services output grows at a lesser rate compared to manufacturing.

Acemoglu and Guerrieri (2008) develop a non-balanced, two-sector growth model without specific references to manufacturing or services. But they document that services are generally more capital intensive than manufacturing. Thus the more (respectively less) capital-intensive sector may be interpreted as services (respectively manufacturing). Similar to Ngai and Pissarides (2008), there is TFP growth, but the ranking of the TFP across the two sectors is not critical to ranking of sectoral growth rates. When TFP growth rates are uniform, output growth rate is faster in the capital-intensive (services) sector. This is termed as capital deepening, which yields that the output and employment of the capital-intensive (services) sector grows faster, compared to manufacturing. The aggregate behavior of the economy is consistent with Kaldor facts.

Whereas the papers cited above refer to consumption services, the most distinguishing feature of our paper is to bring the growth of *business* services to the forefront and show how it may exceed the growth of consumption services and manufacturing.

It purports to explain higher growth of both employment and output in the service sector – rather than one or the other. We emphasize two supply-side factors behind the pattern of differential growth rates among business services, consumption services and manufacturing, namely, differences in returns to scale and labor frictions. Furthermore, in our model differences in growth rates of sectoral outputs tend to persist in the long run, i.e., they do not vanish asymptotically.[12]

3 The Basic Model

The source of growth *per se* is not our central concern. Throughout our analysis, we abstract from TFP growth or physical capital accumulation and assume a simple story of human-capital-accumulation based growth. How growth rates may differ across sectors is our focus.

A closed economy has two sectors: manufacturing (the numeraire sector) and business services. Both sectors are perfectly competitive. Manufacturing output is produced by labor and business services via a decreasing and variable returns to scale technology so as to imply sluggish adjustment in the employment of labor, while business services are produced by labor only under constant returns. More generally, higher returns to scale in the services sector – not necessarily constant returns in that sector and decreasing-returns in manufacturing – would yield the same results.

Difference in returns to scale implies difference in growth rates of sectoral outputs, but not in sectoral employment growth rates. Higher worker frictions in manufacturing (relative to services) would imply that the growth rate of employment in the services sector is higher than that in manufacturing.

3.1 The Static General Equilibrium

Let $q_{st} = L_{st}$ denote the business-service production function, where q_{st} is the total output and L_{st} is the amount of effective labor used in producing business services at time t. Free entry and exit imply the zero-profit condition: $p_{st} = w_t$, where p_{st} is the price of business services. Labor is measured in efficiency units and it grows over time. Its growth process will be specified later, but, at the moment, it is to be noted that w_t is the wage rate per such efficiency unit, *not* earnings per worker per unit of time; see, for instance, Jung and Mercenier (2010).

In the context of the manufacturing sector we keep in view frictional or congestion problems associated with labor size in a firm being large. They manifest in course of working with other factors of production (which give rise to the standard positive but diminishing marginal returns) as well as among workers (such as interpersonal conflicts of various kinds). This leads to a direct loss of output, which is *not* attributable simply to the loss of aggregate labor time available for production. We do not develop a micro structure to incorporate worker frictions in manufacturing and the resultant inflexibility in employment variation. Instead, we postulate that the technology itself features this attribute. Let the production function be:

$$q_{mt} = L_{mt}^{\alpha} q_{st}^{\beta} - \gamma L_{mt}, \ \alpha, \ \beta, \ \gamma > 0, \ \alpha + \beta < 1, \tag{1}$$

where L_{mt} is the effective labor used in manufacturing at time t. The term, $L_{mt}^{\alpha} q_{st}^{\beta}$, may be interpreted as gross output, whereas γL_{mt} can be thought of as a penalty or loss of output because of worker frictions.

The production function (1) satisfies decreasing returns but is non-homothetic. The parameter γ being positive, cost minimization would imply that in response to a proportionate increase in labor and service input costs, the *proportional* reduction in labor employment is less than that of the services input, i.e., labor to services input ratio increases. Likewise, in the face of a proportional decrease in input prices, labor employment is increased less than proportionately compared to the services input, i.e., labor to services input ratio falls. In this sense, γ is the measure of worker frictions and resulting employment inflexibility in manufacturing.

Note that (1) permits negative marginal product – which can be interpreted as a *strong* congestion effect (whereas diminishing but positive returns for any level of employment may be seen as a situation of weak congestion effect). But, profit maximization would imply that in equilibrium the marginal returns to labor must be positive.[13] However, the *possibility* of negative returns has implications for equilibrium where the returns are positive.

The first-order conditions with respect to labor and services input are:

$$\alpha L_{mt}^{\alpha-1} q_{st}^{\beta} = w_t + \gamma^{14} \tag{2}$$

$$\beta L_{mt}^{\alpha} q_{st}^{\beta-1} = p_{st}. \tag{3}$$

The l.h.s. and r.h.s of (2) can be respectively interpreted as the marginal product of labor in producing the *gross* output and the *effective* marginal cost of labor. Using (1), (2) can be stated indirectly as

$$\frac{\alpha q_{mt}}{L_{mt}} = w_t + (1-\alpha)\gamma. \tag{4}$$

Substituting the business services sector relations $q_{st} = L_{st}$ and $p_{st} = w_t$ into the ratio of the two first-order conditions in manufacturing, we get

$$\frac{L_{st}}{L_{mt}} = \frac{\beta}{\alpha} \cdot \frac{w_t + \gamma}{w_t}. \tag{5}$$

It reflects that the ratio of employment between the two sectors is proportional to the ratio of effective marginal costs of hiring labor in the two sectors.

We rewrite the manufacturing production function as eq. (6) below, wherein the production function of the business service sector is substituted . Eq. (7) is the full-employment condition, where $\bar{L}_t$ is the total labor (in effective units) available for production.

$$q_{mt} = L_{mt}^{\alpha} L_{st}^{\beta} - \gamma L_{mt} \tag{6}$$

$$L_{mt} + L_{st} = \bar{L}_t. \tag{7}$$

Static equilibrium is described by eqs. (4)-(7).

Lemma 1 *The static equilibrium exists and is unique for any $\bar{L}_t > 0$.*

Proof: Eqs. (4), (5) and (7) yield

$$L_{mt} = \frac{\alpha w_t}{\alpha w_t + \beta(w_t + \gamma)} \bar{L}_t; \quad L_{st} = \frac{\beta(w_t + \gamma)}{\alpha w_t + \beta(w_t + \gamma)} \bar{L}_t = q_{st}. \tag{8}$$

$$q_{mt} = \frac{w_t[w_t + (1-\alpha)\gamma]}{\alpha w_t + \beta(w_t + \gamma)} \bar{L}_t. \tag{9}$$

If we substitute (8) and (9) into (6),

$$\bar{L}_t = \left(\alpha + \beta + \frac{\beta\gamma}{w_t}\right) \left[\frac{\alpha^{\alpha}\beta^{\beta}}{w_t^{\beta}(w_t + \gamma)^{1-\beta}}\right]^{\frac{1}{1-\alpha-\beta}} \equiv \bar{L}(w_t). \tag{10}$$

The function $\bar{L}(w_t)$ is continuous and differentiable, satisfying $\bar{L}'(w_t) < 0$. Further, $\bar{L}(\cdot) \to 0$ or ∞ as $w_t \to \infty$ or 0. Hence, for any $\bar{L}_t > 0$, a positive solution for w_t exists and it is unique. Eqs. (8) and (9) imply that employment and output solutions are unique. ■

Eq. (10) essentially states that total labor demand is negatively related to the wage rate, which results from decreasing returns to scale in manufacturing. Also recall that w_t in our model is the wage rate per *efficiency unit*. Wage earnings per worker equal $w_t\bar{L}_t$, an increasing function of $\bar{L}_t$.

Consider comparative statics of an increase in $\bar{L}_t$, the stock of effective labor.

Proposition 1 *If $\bar{L}_t$ increases, wage rate (per unit of effective labor) falls, employment and output in both sectors expand, and output and employment ratios, q_{st}/q_{mt} and L_{st}/L_{mt}, both increase.*

Proof: Since $\bar{L}'(w_t) < 0$, w_t falls. Suppose L_{st} falls too. In view of (5), L_{mt} decreases. But both L_{st} and L_{mt} falling as $\bar{L}_t$ increases is incompatible with the full employment equation. Hence L_{st} increases, and thus q_{st} rises too. Eqs. (4) and (6) imply (2), in the light of which a decrease in w_t and an increase in q_{st} imply that L_{mt} increases. As both L_{mt} and q_{st} increase, q_{mt} also increases.

In view of eq. (5), the ratio L_{st}/L_{mt} rises. Using $q_{st} = L_{st}$ and dividing (5) by (4),

$$\frac{q_{st}}{q_{mt}} = \frac{\beta(w+\gamma)}{w_t[w_t + (1-\alpha)\gamma]}.$$

The r.h.s. is a decreasing function of w_t. As w_t falls, the ratio q_{st}/q_{mt} must increase.■

3.2 Households

The economy consists of infinitely lived representative households, who can be treated as one unit. At a given point of time, the representative household possesses L_t units of effective labor and one unit of time. It could spend its time in either augmenting its human capital or working in production sectors. Let $H_t \in (0,1)$ denote time in human capital investment and let

$$L_{t+1} = a_L H_t L_t, \quad a_L > 1. \tag{11}$$

Thus the growth rate of human capital is proportional to the time invested in human capital. Since there are no education sectors, eq. (11) can be seen as a self-learning function. The trade-off is that the higher the investment in human capital, the greater will be the effective labor and hence the higher will be the total wage earnings in the future, but the less will be the total wage earnings in the current period.

There are two sources of income: wage income in both sectors and profit income in manufacturing (π_m). In making consumption choices, these incomes are treated as exogenous by a household.

Denoting the discount factor by ρ, the amount consumed of manufacturing by c_{mt} and assuming the felicity function $\ln c_{mt}$, we can write down the household's problem as:

$$\text{Maximize} \sum_{t=0}^{\infty} \rho^t \ln c_{mt}, \text{ subject to (11), and the budget } c_{mt} \leq w_t \bar{L}_t + \pi_{mt},$$

where $\bar{L}_t \equiv (1 - H_t)L_t$ is the total effective labor working in the production sectors. Given L_0, the household chooses $\{c_{mt}\}_0^\infty$, $\{H_t\}_0^\infty$ and $\{L_t\}_1^\infty$.

The Euler equation and the transversality conditions are:

$$\frac{c_{mt+1}/w_{t+1}}{c_{mt}/w_t} = \rho a_L \tag{12}$$

$$\lim_{t\to\infty} \frac{\rho^t w_t L_{t+1}}{a_L c_{mt}} = 0. \tag{13}$$

We assume $\rho a_L > 1$, such that the c_{mt}/w_t ratio grows at a positive rate. A marginal increase in investment entails a marginal loss in terms of current utility equal to w_t/c_{mt} and entitles a marginal gain in terms of future utility equal to $a_L w_{t+1}/c_{mt+1}$. At the optimum, the former is equal to the discounted value of the latter.[15]

3.3 Dynamics

A perfect-foresight, dynamic, competitive equilibrium is a set of sequences of $\{w_t\}_0^\infty$, $\{p_{st}\}_0^\infty$, $\{\pi_{mt}\}_0^\infty$, $\{q_{mt}\}_0^\infty$, $\{q_{st}\}_0^\infty$, $\{L_{mt}\}_0^\infty$, $\{L_{st}\}_0^\infty$, $\{c_{mt}\}_0^\infty$, $\{H_t\}_0^\infty$ and $\{L_t\}_1^\infty$, such that

(i) $\{c_{mt}\}_0^\infty$, $\{H_t\}_0^\infty$ and $\{L_t\}_1^\infty$ solve the household problem, given $\{w_t\}_0^\infty$, $\{\pi_{mt}\}_0^\infty$ and the initial condition L_0,

(ii) $c_{mt} = q_{mt}$ (market clearing),

where, from the static equilibrium, w_t, p_{st}, π_{mt}, q_{st}, L_{mt} and L_{st} are implicit functions of $\bar{L}_t \equiv (1 - H_t)L_t$.

We have

Proposition 2 *Output and employment in both sectors grow, and, the growth rates of output and employment in the business services sector are higher than those in the manufacturing sector.*

Proof: Substituting $c_{mt} = q_{mt}$ into the Euler equation, we see that the q_{mt}/w_t ratio grows at the (gross) rate ρa_L. If we substitute (4)–(5) into the full-employment equation (7), we have

$$\frac{q_{mt}}{w_t} = \frac{\bar{L}_t[1 + \Phi(\bar{L}_t)]}{\alpha + \beta}, \text{ where } \Phi(L) \equiv \frac{\gamma\alpha(1 - \alpha - \beta)}{(\alpha + \beta)w(\bar{L}_t) + \beta\gamma} > 0. \tag{14}$$

We have $\Phi'(\cdot) > 0$, since $w'(\bar{L}_t) < 0$. Thus q_{mt}/w_t bears an increasing, one-to-one, relation with $\bar{L}_t$. Hence $\bar{L}_t$ grows over time. In view of Proposition 1, output and employment in both sectors grow; the proportions q_{st}/q_{mt} and L_{st}/L_{mt} rise, implying that growth rates of output and employment in the business-services sector exceed those in manufacturing.[16] ■

Importantly, note that if the friction parameter γ were zero, the output of the business service sector would still grow faster than that of manufacturing, but the employment growth in the two sectors will be the same. Hence, unbalanced growth of sectoral outputs follows from difference in returns to scale and that of employment stems from differences in worker frictions across the two sectors. Proposition 2 is consistent with our stylized fact insofar as it compares the business-services sector to manufacturing.

Ours is a one-factor model without physical capital, so compliance with many Kaldor facts is outside its purview. However,

Proposition 3 *As $t \to \infty$, per capita real income tends to grow at a constant rate.*

Proof: Eq. (14) and that q_{mt}/w_t grows at a constant rate for all t imply that $\lim_{t\to\infty} \bar{L}_t = \infty$. Hence, from (10), $\lim_{t\to\infty} w_t = 0$. In view of (14), $\lim_{t\to\infty} q_{mt}/w_t \propto \bar{L}_t$; thus the growth rate of $\bar{L}_t$ approaches ρa_L. Consider (8). We have $L_{st} \simeq \bar{L}_t$, since $w_t \to 0$. Hence L_{st}, and thus q_{st} approach the growth rate, ρa_L. From (2), it follows that the growth rate of L_{mt} tends to $(\rho a_L)^{\beta/(1-\alpha)}$. In view of (4), $\lim_{t\to\infty} q_{mt} \propto L_{mt}$. Hence, the growth rate of q_{mt} approaches $(\rho a_L)^{\beta/(1-\alpha)}$.

Since population is fixed, per capita income, proportional to aggregate income, q_{mt}, tends to grow at $(\rho a_L)^{\beta/(1-\alpha)}$. ■

Dynamics of Learning and the Transversality Condition

The solution of the dynamic model is not complete without characterizing the dynamics of investment in human capital, H_t. It will be shown in Appendix A that $H_t < \rho$ for all t and approaches ρ. Moreover, along the solution path, the transversality condition (13) is met.

4 Services for Households

The basic model is now extended to include household or consumer services. It is the main section of this paper. Unlike Buera and Kaboski (2012), all such services are provided by the market. The services sector has two competitive sub-sectors: business services and consumer services. The resulting model implies the stylized fact that in terms of both output and employment, the growth rates of the business services sub-sector exceed those of the consumer services sub-sector, which, in turn, are higher than those of the manufacturing sector.

We assume here that business and household services are distinct: one set of services are demanded mostly by businesses (manufacturing) and the other by households. It will be shown in Section 5 that similar conclusions hold for services shared by businesses and households.

The behavior of the business-service providers is the same as before. Let the household-service providers face similar constant-returns technology. For algebraic simplicity, we use the same production function: $q^h_{st} = L^h_{st}$. (A single firm may provide both.)

Households derive utility from consuming the manufacturing good as well as consumer services. Let the felicity function be $U_t = \lambda \ln c_{mt} + (1-\lambda)\ln c^h_{st}, \lambda \in (0,1)$, where c^h_{st} is the quantity of consumer services demanded. The assumed utility function implies that the income elasticity of demand for either good is unity; this will be relaxed in Section 5.

The household's problem is to maximize $\sum_{t=0}^{\infty} \rho^t U_t$, subject to the learning function (11) and the budget $c_{mt} + p^h_{st} c_{st} \leq w_t \bar{L}_t + \pi_{mt}$, where p^h_{st} is the price of consumer services. The dichotomy between the static and the dynamic components of the household's optimization problem is obvious. The former yields

$$\frac{\lambda}{1-\lambda} \frac{c^h_{st}}{c_{mt}} = \frac{1}{p^h_{st}}. \tag{15}$$

In the supply side, zero-profit conditions of service firms are:

$$p_{st} = p^h_{st} = w_t. \tag{16}$$

The situation of the manufacturing sector is same as in the basic model. Eqs. (4)–(6) continue to hold. In equilibrium, $c_{mt} = q_{mt}$ and $c^h_{st} = q^h_{st} = L^h_{st}$. Substituting these and the zero-profit conditions (16) into the first-order condition (15) gives the analog of (5) for the household:

$$\frac{\lambda}{1-\lambda} \cdot \frac{L^h_{st}}{q_{mt}} = \frac{1}{w_t}. \tag{17}$$

Finally, we have the full-employment condition:

$$L_{mt} + L_{st} + L^h_{st} = \bar{L}_t. \tag{18}$$

Eqs. (4)–(6) together with (17)–(18) constitute the static production system of the economy. They determine five variables: the wage rate in the economy, employment and output in manufacturing and those in the two service sub-sectors. Various substitutions lead to an analog of (10):

$$\bar{L}_t = \left\{ \alpha + \beta + \frac{\beta\gamma}{w_t} + \frac{1-\lambda}{\lambda}\left[1 + \frac{(1-\alpha)\gamma}{w_t} \right] \right\} \left[\frac{\alpha^\alpha \beta^\beta}{w_t^\beta (w_t+\gamma)^{1-\beta}} \right]^{\frac{1}{1-\alpha-\beta}} \equiv \tilde{L}(w_t). \quad (19)$$

A solution to this equation exists and it is unique and it implies the same for other variables. Lemma 1 thus holds. We have $\tilde{L}'(w_t) < 0$ and as $\bar{L}_t \to 0$ or ∞, w_t approaches ∞ or 0. As an extension of Proposition 1,

Proposition 4 *An increase in $\bar{L}_t$ leads to a decrease in the wage rate (per unit of effective labor); expansions in output and employment in manufacturing and the two service sub-sectors, and increases in*

$$\frac{L_{st}^h}{L_{mt}}; \quad \frac{q_{st}^h}{q_{mt}}; \quad \frac{L_{st}}{L_{st}^h}; \quad \frac{q_{st}}{q_{st}^h}.$$

Proof: In view of (19), w_t falls. It is straightforward to derive that L_{mt}, L_{st} and L_{st}^h all increase. Hence, employment and output expand in each sector or sub-sector. Multiplying (4) by (17) yields

$$\frac{L_{st}^h}{L_{mt}} = \frac{1-\lambda}{\alpha\lambda} \cdot \frac{w_t + (1-\alpha)\gamma}{w_t}, \quad (20)$$

the r.h.s. of which is a decreasing functions of w_t. Hence, the L_{st}^h/L_{mt} ratio rises as w_t falls. In view of (17), q_{st}^h/q_{mt} rises, since $q_{st}^h = L_{st}^h$. Next, divide (5) by (20). It gives

$$\frac{L_{st}}{L_{st}^h} = \frac{\beta\lambda}{1-\lambda} \cdot \frac{w_t + \gamma}{w_t + (1-\alpha)\gamma}. \quad (21)$$

The r.h.s. increases as w_t falls; hence this ratio rises. Given $q_{st} = L_{st}$ and $q_{st}^h = L_{st}^h$, the q_{st}/q_{st}^h ratio also increases. ■

The dynamic part of the household optimization remains essentially same. The ratio of total household expenditure to the wage rate grows at the rate ρa_L. Since the expenditure on manufacturing constitutes a constant fraction (λ) of total household expenditure, the Euler equation (12) continues to hold.[17] The ratio q_{mt}/w_t grows at the constant rate ρa_L. Similar to the basic model, the static system implies

$$\frac{q_{mt}}{w_t} = \frac{\lambda \bar{L}_t[1+\bar{\Phi}(\bar{L}_t)]}{1-\lambda(1-\alpha-\beta)}, \text{ where} \tag{22}$$
$$\bar{\Phi}(\bar{L}_t) \equiv \frac{\lambda\alpha\gamma}{[1-\lambda(1-\alpha-\beta)]w(\bar{L}_t)+[(1-\lambda)(1-\alpha)+\lambda\beta]\gamma}$$

and it has the same limit properties. As $\bar{\Phi}(\cdot)$ is increasing in $\bar{L}_t$, $\bar{L}_t$ grows over time without bound.[18]

Our central position below – which ranks growth rates within the services sector vis-a-vis manufacturing – follows immediately in the light of Proposition 4.

Proposition 5 *The output and employment in the business services sub-sector grow faster than output and employment (respectively) in the consumer services sub-sector, which, in turn, grow faster than output and employment (respectively) in manufacturing.*

The upshot is that the employment and output growth rankings among the two service sub-sectors and manufacturing accord with the stylized fact we wish to explain. To understand this intuitively, it will be useful to first think what the ranking would have been if worker frictions in manufacturing were absent. It is clear that employment would grow at the same rate in all the three 'sectors.' Because the technology is similar between the two service sub-sectors, their outputs would have grown at the same rate. This common rate would have exceeded the growth rate of manufacturing, because returns to scale are lower in manufacturing.

Now bring into consideration the presence of worker frictions in manufacturing.

They would imply a relatively higher demand for business services and less for labor as manufacturing output expands. Hence, compared to the case of no worker frictions in manufacturing, the growth rate of employment in the business-service sub-sector would exceed that in manufacturing, while the growth rate of employment in the consumer-services sub-sector would lie in-between. The same ranking extends to output growth rates.

Furthermore, as in the basic model, the growth rate of per capita real income is bounded away from zero and approaches a constant rate, i.e., Proposition 3 holds. This is proved in Appendix B.

5 Generalizations and Alternative Environments

Main results obtained in the preceding sections are robust to some generalizations and alternative market environments.

5.1 Service-Oriented Relative Demand Shift

The relative rise of the service sector in the post-WWII era has been largely attributed to the hypothesis that as real income rises the consumer demand for services rises more than proportionately, i.e., the income elasticity of demand for household services exceeds one.

It is shown below that such a preference structure, which leads to a relative demand shift towards consumer services, tend to imply higher growth rates of output and employment in the household services sub-sector. Hence the growth ranking between the two service sub-sectors becomes ambiguous, while that between the services sector as a whole and manufacturing remains in tact.

Let a household's felicity function be $U_t = \lambda \ln c_{mt} + (1-\lambda)\ln(c^h_{st} + \delta), \lambda \in (0,1),\ \delta > 0$. The presence of the parameter δ, an index of 'non-essentiality' of services in consumption, implies income elasticity of demand for consumer services to be greater than unity. Static optimization has the first-order condition

$$\frac{\lambda}{1-\lambda}\frac{c_{st}^h+\delta}{c_{mt}}=\frac{1}{p_{st}^h}. \tag{23}$$

All other equations remain the same as in Section 4 except (17), which is replaced by

$$\frac{\lambda}{1-\lambda}\cdot\frac{L_{st}^h+\delta}{q_{mt}}=\frac{1}{w_t}. \tag{24}$$

This follows from (23) by substituting $c_{st}^h=q_{st}^h=L_{st}^h$ and $p_{st}^h=w_t$.

Appendix C works out the solution of the static system. Qualitatively, the effects of an increase in $\bar{L}_t$ on the wage rate and sectoral employment and outputs are same as earlier.

The nature of dynamic trade-off for the household is also the same. By substituting (24) into the budget constraint and eliminating c_{st}^h, it can be derived that the c_{mt}/w_t ratio grows at the rate ρa_L. Hence q_{mt}/w_t – and thus $\bar{L}_t$ – grow over time.

Output and employment growth rate rankings are given by

Proposition 6 *In the presence of income-induced relative demand shift towards consumer services, output as well as employment growth rates in business and consumer services sub-sectors cannot be ranked, but both growth rates exceed those in manufacturing.*

Proof: From (4) and (5) and that w_t decreases over time, it follows that the business services output (respectively employment) grows more rapidly than manufacturing output (respectively employment). Likewise, in view of (4), (24) and w_t falling over time, the consumer-services output (respectively employment) also rises faster than manufacturing output (respectively employment).

Eliminating q_{mt} and L_{mt} from (4), (5) and (24) yields

$$\frac{\beta\lambda}{1-\lambda}\cdot\frac{L_{st}^h+\delta}{L_{st}}=\frac{w_t+(1-\alpha)\gamma}{w_t+\gamma}.$$

Since w_t decreases over time, the r.h.s. falls and thus $(L^h_{st} + \delta)/L_{st}$ declines with time. But δ being positive, the ratio L^h_{st}/L_{st}, equal to q^h_{st}/q_{st}, may increase or decrease over time. ■

The relation between the manufacturing sector and the business service firms is the same as in the previous model; hence the growth-rate rankings between them is the same. The demand shift towards consumer services constitutes an added factor for its growth. Hence its growth rate remains higher than that in manufacturing.

However, growth rates between the two sub-sectors within the services sector cannot be unambiguously ranked, because, on one hand, business services tend to grow faster than consumption services due to labor frictions in manufacturing, while, on the other hand, because of the relative demand shift towards consumption services, consumption-service production would tend to grow faster than business services. It depends on the magnitudes of labor friction in manufacturing (γ) and the degree of non-essentiality of consumption services (δ), relative to each other.

In what follows, we revert back to the assumption of homothetic preferences, as in the base model.

5.2 Services Shared by Businesses and Households

We have considered business and consumer services as distinct products. There are however many types of services demanded by both businesses and households. Examples include retail trade, transport and communication and financial intermediation. Consider the scenario where the same service is sold to firms in the manufacturing sector as well as to households. It will be shown that the same rankings between growth rates of manufacturing and volumes of (same) services sold to the 'two sectors' hold, as was true for pure business and consumer services.

Let the common price for the service good be denoted as p^c_{st} and the production function be

$$q_{st} + q_{st}^h = L_{st}^c. \tag{25}$$

As earlier, competitive pressures imply $p_{st}^c = w_t$.

Relations pertaining to the manufacturing sector and households are unchanged. The model structure follows that in Section 4, except that there is no notion of L_{st} or L_{st}^h; they are substituted respectively by q_{st} and q_{st}^h. Similar to Section 4, an increase in $\bar{L}_t$ implies a decline in w_t and increases in q_{st}, q_{st}^h, L_{mt} and q_{mt}; increases in q_{st} and q_{st}^h imply that L_{st}^c increases, i.e., employment expands in the services sector too.

The Euler equation remains same; thus q_{mt}/w_t grows at the gross rate of ρa_L. This implies that $\bar{L}_t$ grows over time. Thus w_t falls, and output and employment in both sectors expand. Furthermore, the ratios L_{st}^c/L_{mt}, q_{st}^h/q_{mt} and q_{st}/q_{st}^h rise over time. Hence,

Proposition 7 *Employment growth is higher in the services sector. In terms of output/sales, the business-oriented component of the services grows faster than the component serving the households services and the latter grows faster than the manufacturing sector.*[19]

5.3 Differentiated Services

Services have been thought of and modeled by many authors as differentiated brands produced in a monopolistically competitive market, e.g., Eswaran and Kotwal (2002) and Matsuyama (2013) among many others. Let q_{st} and c_{st}^h denote respectively composites of business and household services, defined respectively by:

$$q_{st} = \left(\int_0^{N_t} q_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1$$

$$c_{st}^h = \left(\int_0^{N_t^h} c_{it}^{h\,\frac{\sigma^h-1}{\sigma^h}} di \right)^{\frac{\sigma^h}{\sigma^h-1}}, \quad \sigma^h > 1,$$

where N_t and N_t^h are the number of respective varieties available at time t, and, σ and σ^h are respectively the elasticity of substitution between any two brands of business and consumer services.

Let both types of services be produced by linear, increasing-returns technologies: $q_{it} = L_{it} - 1$ for business services and $q_{it}^h = L_{it}^h - 1$ for household services. As long as manufactures are produced by the same decreasing-returns to scale technology (1), the same qualitative differences between the sectors continue to hold.

As shown in a previous version of this paper, all results hold under one further assumption, namely,

$$\alpha + \frac{\beta\sigma}{\sigma - 1} < 1. \tag{26}$$

Increasing returns to scale along with constant marginal product of labor in the production of services present an element of instability in the labor market. The inequality (26) is indeed a stability condition for the labor market.[20]

5.4 Manufacturing as an Input in the Production of Services

Production of services typically uses products, tools and equipment from manufacturing, both as durables and intermediates. For instance, transportation services use capital goods like vehicles. Financial services extensively require computers and modern tools of information technology. Almost all services use a variety of "consumables" produced in the manufacturing sector. However, physical capital accumulation is beyond the scope our analysis. It is shown that the growth ranking between the two sectors remains the same even if services production required manufactures as intermediates.

For simplicity of illustration, we consider business services only. Let the production function of the business-services sector be:

$$q_{st} = L_{st}^{\eta} {q'}_{mt}^{1-\eta}, \quad 0 < \eta < 1, \tag{27}$$

where q'_{mt} is the manufacturing input. The first-order conditions can be stated as

$$\eta p_{st} q_{st} = w_t L_{st}, \tag{28}$$

$$\frac{L_{st}}{q'_{mt}} = \frac{\eta}{1-\eta}\frac{1}{w_t}. \tag{29}$$

The price of services is no longer proportional to the wage rate. From (27)-(29), it can be derived that $p_{st} \propto w_t^\eta$. The manufacturing firm's problem is same as in the base model. Eq. (28) and the cost-minimization condition in manufacturing imply

$$\frac{\alpha}{\beta\eta}\frac{L_{st}}{L_{mt}} = \frac{w_t + \gamma}{w_t}. \tag{30}$$

The static general equilibrium is spelt by (1)-(2), the full employment condition (7) and (27)– (30). The same comparative statics hold: an increase in $\bar{L}_t$ leads to a decrease in w_t and increases in sectoral output and employment levels. In view of (30), as the wage rate falls, the ratio of employment in business services to that in manufacturing increases. Using $p_{st} \propto w_t^\eta$ and eqs. (1)-(2), eqs. (28) and (30) yield

$$\frac{q_{st}}{q_{mt}} \propto \frac{1}{w_t^\eta}\left[1 + \frac{\alpha\gamma}{w_t + (1-\alpha)\gamma}\right].$$

Hence, the services output to manufacturing output ratio rises.

The household's problem is the same as in the base model. The Euler equation states that c_{mt}/w_t ratio grows at the constant rate ρa_L. Substituting (1)-(2), (7) and (29)–(30) into the manufacturing market clearing condition $c_{mt} = q_{mt} - q'_{mt}$,

$$\frac{c_{mt}}{w_t} = \frac{(1-\beta+\beta\eta)\bar{L}_t}{\alpha+\beta\eta}\left[1 + \frac{\alpha\gamma(1-\alpha-\beta)}{(1-\beta+\beta\eta)[\beta\eta\gamma + (\alpha+\beta\eta)w(\bar{L}_t)]}\right], \tag{31}$$

Given that $w'(\bar{L}_t) < 0$ and c_{mt}/w_t grows without bound, $\lim_{t\to\infty} \bar{L}_t = \infty$ and $\lim_{t\to\infty} w_t = 0$. The growth rate of $\bar{L}_t$ asymptotes to ρa_L.

The next proposition states the sectoral output and employment rankings as well as how growth rates and the differences in growth rates are sensitive to the share of manufacturing in services.

Proposition 8 *(a) Output and employment growth rates in the business services sector are higher than those in manufacturing. (b) In the long run, the higher the share of manufacturing in the business-services sector, the slower are the output growth rates of both sectors, the larger the gap in the employment growth rates and the smaller is the gap in the output growth rates.*

Proof: Part (a) is obvious. Following the same logic and algebraic manipulations as in the base model, the asymptotic growth rates of sectoral employment and output can be calculated as:

$$\text{(i) } \frac{L_{st+1}}{L_{st}} \to \rho a_L; \text{ (ii) } \frac{L_{mt+1}}{L_{mt}} \to (\rho a_L)^{\frac{\eta\beta}{\eta\beta+1-\alpha-\beta}};$$
$$\text{(iii) } \frac{q_{st+1}}{q_{st}} \to (\rho a_L)^{\frac{\eta(1-\alpha)}{\eta\beta+1-\alpha-\beta}}; \text{ (iv) } \frac{q_{mt+1}}{q_{mt}} \to (\rho a_L)^{\frac{\eta\beta}{\eta\beta+1-\alpha-\beta}}.$$

The expression (i) is independent of η, whereas (ii)-(iv) are increasing functions of η. A decrease in η reflects a higher share of manufacturing input in services production. It follows that $L_{st+1}/L_{st} - L_{mt+1}/L_{mt}$ increases and $q_{st+1}/q_{st} - q_{mt+1}/q_{mt}$ decreases as η falls – which proves Part (b). ■

It is interesting that as the share of manufacturing in services production increases, the growth gap between the two sectors in terms of employment increases, but that in terms of output falls.

Intuitively, the growth of the total supply of effective labor depends on the rate of time discount (ρ) and the productivity in the enhancement of human capital (a_L), not on η. Since employment growth in the services sector is higher, in the long run it tows that of aggregate supply of effective labor, hence independent of η. At the same time, the dependence of technology of producing business-services on manufacturing goods as inputs has a 'locomotive' effect: a slower growing sector's output being used as input in the faster growing sector, the growth rate of the latter is pulled down, which, in turn, drags down the growth rate in the former sector. Hence, the employment growth rate of manufacturing as well as output growth rates in both sectors decline as η falls. It then follows that, as η declines, the difference between the output growth rates falls, while the difference between the employment growth rates rises (because the growth rate of employment in the services sector is independent of η).[21]

6 Concluding Remarks

In the post WWII world economy the services sector has grown consistently faster than manufacturing. In many countries the share of this sector in GDP now stands well above 50%. This phenomenon has been mainly attributed to a relative demand shift towards consumer services as real income rises. While this may very well be true, we have taken the position that it is not designed to explain the growth of *business* services in particular. We have posited a stylized fact that business services have grown faster than consumer services, which, in turn, have outpaced manufacturing.

Our analysis began with business services, and consumption services were introduced later. We believe it has enabled us to uncover some supply-side factors behind the rise of the services sector relative to manufacturing. One is higher returns to scale in the services sector compared to manufacturing, although in our model we have assumed a specific structure. Prevalence of worker frictions in manufacturing (relative to services) is another. In tandem, these two factors explain the stylized fact.

By abstracting from TFP growth, the general goal of our analysis is to understand *inter*-sectoral – rather than *intra*-sectoral or intra-sub-sectoral – differences in the growth rates of employment and output. However, major productivity improvements have been recorded not just for manufacturing but also in the services sector. Triplett and Bosworth (2003) noted that the TFP growth in the services sector is no less than that in manufacturing.[22] Heshmati (2003) presents a survey of productivity growth in many manufacturing and services industries. He notes that over time services productivity has grown over time, and, owing to services outsourcing, has contributed to higher productivity growth in manufacturing.

We have incorporated a very simple source of growth namely that of human capital. The static implications of an increase in overall resources available to an economy map directly to growth rates. Current research under way by the authors incorporates TFP growth, capital accumulation as well as returns to scale differences. The resulting analysis generates ranking of services, manufacturing *and* agriculture in decreasing order in terms of both output and employment growth. The model also complies with Kaldor facts.

Also, growth of services outsourcing by the manufacturing sector is a part of overall growth of business services and has become common to many economies. Future research must address this. Another endeavor will be to analyze the so-called second wave of the burgeoning share of the services sector in an aggregate economy by incorporating computer capital and IT infrastructure.

Last but not least, whereas our analysis is confined to a closed economy, it is important to introduce international trade – in goods *and* services – which would permit to analyze the growth of the services sector in the context of the global economy.

Appendix A

It refers to Section 3.

Growth Rate of $\bar{\mathbf{L}}_{\mathbf{t}}$

Lemma A1: $g_{\bar{L}_t} \equiv \dfrac{\bar{L}_{t+1}}{\bar{L}_t} < \rho a_L$.

Proof: Since q_{mt}/w_t grows at the rate of ρa_L, eq. (14) implies

$$g_{\bar{L}_t} = \rho a_L \cdot \frac{1 + \Phi(\bar{L}_t)}{1 + \Phi(\bar{L}_{t+1})}. \tag{A.1}$$

Further, $\bar{L}_{t+1} > \bar{L}_t$ (as $\bar{L}_t$ increases over time) and $\Phi'(\cdot) > 0$ imply $\Phi(\bar{L}_t) < \Phi(\bar{L}_{t+1})$. Hence, $g_{\bar{L}_t} < \rho a_L$. ■

Lemma A2: $g_{\bar{L}_t} \to \rho a_L$ as $\bar{L}_t \to 0$ or ∞.

Proof: As $\bar{L}_t \to 0$ or ∞, $w_t \to \infty$ or 0 and hence the term in the square brackets of (14) approaches 1 or $1 + (\alpha/\beta)(1 - \alpha - \beta)$. In either case, $q_{mt}/w_t \propto \bar{L}_t$. Hence $g_{\bar{L}_t} \to \rho a_L$.[23] ■

Dynamics of $\mathbf{H}_{\mathbf{t}}$

By using $\bar{L}_t \equiv (1 - H_t)L_t$ and the learning function (11),

$$\Delta H_t \equiv H_{t+1} - H_t = (1 - H_t)\left(1 - \frac{g_{\bar{L}_t}}{a_L H_t}\right). \tag{A.2}$$

Hence $\Delta H_t = 0$ spells the relation

$$H_t = \frac{g_{\bar{L}_t}}{a_L} \equiv \Psi(\bar{L}_t), \tag{A.3}$$

where $\Psi(\cdot)$ is an implicit function based on (A.1).

Figure 1 around here.

In view of Lemmas A1 and A2 and the continuity and differentiability of the function $\Psi(\cdot)$, $\Psi(\cdot) < \rho$ for any $\bar{L}_t > 0$ and $\Psi' \lesseqgtr 0$ as $\bar{L}_t \to 0$ or ∞.

Consider Figure 1, which depicts the function $\Psi(\cdot)$, same as $\Delta H_t = 0$, and, the dynamics of H_t and $\bar{L}_t$.[24] We have $\Delta H_t \gtrless 0$ according as $(\bar{L}_t,\, H_t)$ lies above or below this curve. This implies the directions of vertical arrows. Because $\bar{L}_t$ increases over time monotonically, the horizontal arrows always point to the right.

There is no steady state in that there is no stationary solution of H_t in (A.3); for any initial value of L_t, H_t varies over time. Any trajectory with an initial value of $H_t \geq \rho$ approaches $H_t = 1$. This would imply $\bar{L}_t \to 0$, which is implausible and inconsistent with $\bar{L}_t$ growing over time. In the domain of $H_t < \rho$, as shown, one set of trajectories approach $H_t = 0$. This would imply the stock of effective labor approaching zero, which is also implausible as well as inconsistent with that $\bar{L}_t$ grows over time. However, there is one, a saddle path, along which H_t asymptotes towards ρ. It will be shown below that this trajectory meets the transversality condition. Hence, it is the solution path, and along this path, $H_t < \rho\ \forall t$ and $\lim\limits_{t\to\infty} H_t = \rho$.

Transversality Condition

By using (9), (11), the market clearing condition $c_{mt} = q_{mt}$ and $\bar{L}_t = (1 - H_t)L_t$,

$$\begin{aligned}\frac{w_t L_{t+1}}{a_L c_{mt}} &= \frac{w_t H_t L_t}{q_{mt}} \\ &= \frac{H_t}{1 - H_t} \cdot \frac{w_t \bar{L}_t}{q_{mt}} \\ &= \frac{H_t}{1 - H_t} \cdot \frac{\alpha w_t + \beta(w_t + \gamma)}{w_t + (1 - \alpha)\gamma}.\end{aligned}$$

Along the saddle path, $\lim_{t\to\infty} H_t/(1 - H_t) = \rho/(1 - \rho)$. As shown earlier, $\lim_{t\to\infty} w_t = 0$. Therefore,

$$\lim_{t\to\infty} \frac{\rho^t w_t L_{t+1}}{a_L c_{mt}} = \frac{\rho\beta}{(1 - \rho)(1 - \alpha)} \cdot \lim_{t\to\infty} \rho^t = 0,$$

i.e. the transversality condition is met along the saddle path.

Appendix B

It refers to Section 4. It will be shown that growth of per capita real income asymptotes a constant rate. Normalizing population size to unity, real per capita income has the expression $I_t^R = (q_{mt} + p_{st}^h q_{st}^h)/(p_{st}^h)^{1-\lambda} \propto q_{mt}^\lambda {L_{st}^h}^{1-\lambda}$ in view of (15)–(17).

The following holds as $t \to \infty$. We have $w_t \to 0$. This implies that the growth rate of $\bar{L}_t$ approaches ρa_L, and $L_{st} \propto \bar{L}_t$, $L_{st}^h \propto L_{st}$ and $q_{mt} \propto L_{mt}$. Thus,

$$\begin{aligned}&\frac{L_{st+1}^h}{L_{st}^h} \to \rho a_L; \quad \frac{q_{mt+1}}{q_{mt}} \to (\rho a_L)^{\frac{\beta}{1-\alpha}} \\ \Rightarrow \frac{I_{t+1}^R}{I_t^R} &= \left(\frac{q_{mt+1}}{q_{mt}}\right)^\lambda \cdot \left(\frac{L_{st+1}}{L_{st}}\right)^{1-\lambda} \\ &\to (\rho a_L)^{\frac{\lambda\beta}{1-\alpha}} (\rho a_L)^{1-\lambda} \\ &= (\rho a_L)^{\frac{\beta\lambda}{1-\alpha} + 1 - \lambda}.\end{aligned}$$

$\blacksquare$

Appendix C

It refers to Section 5.1, which introduces relative demand shift towards consumer services. The static system is characterized by (4)–(6), the full-employment condition (18), and (24). Eliminating the variables q_{mt}, L_{mt}, L_{st} and L_{st}^h, the following equation summarizes the static equilibrium in terms of solving w_t.

$$(\alpha^\alpha \beta^\beta)^{\frac{1}{1-\alpha-\beta}} \left[\frac{1-\lambda}{\lambda}\Omega(w_t) + \Gamma(w_t)\right] - \delta = \bar{L}_t \tag{A.4}$$

where

$$\Omega(w_t) \equiv \frac{w_t + (1-\alpha)\gamma}{w^{\frac{1-\alpha}{1-\alpha-\beta}}(w_t+\gamma)^{\frac{1-\beta}{1-\alpha-\beta}}}; \quad \Gamma(w_t) \equiv \frac{(\alpha+\beta)w_t + \beta\gamma}{w^{\frac{1-\alpha}{1-\alpha-\beta}}(w_t+\gamma)^{\frac{1-\beta}{1-\alpha-\beta}}}.$$

Both $\Omega'(w_t)$ and $\Gamma'(w_t)$ being negative, an increase in $\bar{L}_t$ implies a fall in w_t. It is straightforward to derive that L_{mt}, L_{st}, L_{st}^h and q_{mt} all increase with $\bar{L}_t$.

References

Acemoglu, Daron and Veronica Guerrieri (2008), "Capital Intensity and Non-Balanced Endogenous Growth," *Journal of Political Economy*, Vol. 116, pp. 467–98.

Basu, Susanto, John G. Fernald, and Miles Kimball (2006), "Are Technology Improvements Contrationary?" *American Economic Review*, Vol. 96, No. 5, pp. 1418–1448.

Baumol, William J. (1967), "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crises," *American Economic Review*, Vol. 57, No. 3, pp. 415–426.

Berry, Ralph E., Jr. (1967), "Returns to Scale in the Production of Hospital Services," *Health Services Research*, Vol. 2, No. 2, pp. 123–139.

Bertola, Giuseppe (1992), "Labor Turnover Costs and Average Labor Demand," *Journal of Labor Economics*, Vol. 10, pp. 389–411.

Beyers, William B. and David P. Lindahl (1996), "Explaining the Demand for Producer Services: Is Cost-Driven Externalization the Major Factor?" *Papers in Regional Science*, Vol. 75, No. 3, pp. 351–374.

Buera, Francisco J. and Joseph P. Kaboski (2012), "The Rise of the Service Economy," *American Economic Review*, Vol. 102, No. 6, pp. 2540–2569.

Cosar, A. Kerem, Nezih Guner, and James Tybout (2010), "Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy," NBER Working Paper No. w16326.

Das, Sanghamitra and Ramprasad Sengupta (2004), "Projection Pursuit Regression and Disaggregate Productivity Effects: The Case of the Indian Blast Furnaces," *Journal of Applied Econometrics*, Vol. 19, No. 3, pp. 397–418.

Duarte, Margarida and Diego Restuccia (2010), "The role of the structural transformation in aggregate productivity," *The Quarterly Journal of Economics*, Vol. 125, No. 1, pp. 129–173.

Eichengreen, Barry and Poonam Gupta (2009), "The Two Waves of Service Sector Growth." NBER Working Paper No. 14968.

——— (2011), "The Service Sector as India's Road to Economic Growth." NBER Working Paper No. 16757.

Business Line (2012), "India can sustain 3% current account deficit: Montek." Available at http://www.thehindubusinessline.com/industry-and-economy/economy/article3404533.ece.

Eswaran, Mukesh and Ashok Kotwal (2002), "The Role of the Service Sector in the Process of Industrialization," *Journal of Development Economics*, Vol. 68, pp. 401–420.

European Research Area (2013), "Employment Protection, Productivity, Wages and Jobs in Europe." European Policy Brief dated 16/2/13. Available at http://ec.europa.eu/research/social-sciences/pdf/policy-briefs-indicser-02-2013_en.pdf.

Fisher, Allan G. B. (1939), "Primary, Secondary and Tertiary Production," *Economic Record*, Vol. 15, No. 1, pp. 24–38.

Guner, Nezih, Gustavo Ventura, and Yi Xu (2008), "Macroeconomic implications of size-dependent policies," *Review of Economic Dynamics*, Vol. 11, No. 4, pp. 721–744.

Heshmati, Almas (2003), "Productivity Growth, Efficiency and Outsourcing in Manufacturing and Service Industries," *Journal of Economic Surveys*, Vol. 17, No. 1, pp. 79–112.

Hubacek, Klaus and Stefan Giljum (2003), "Applying Physical Input-Output Analysis to Estimate Land Appropriation (Ecological Footprints) of International Trade Activities," *Ecological Economics*, Vol. 44, No. 1, pp. 137–151.

Jung, Jaewon and Jean Mercenier (2010), "Routinization-Biased Technical Change, Globalization and Labor Market Polarization: Does Theory Fit the Facts?". Mimeo.

Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie (2001), "Beyond Balanced Growth," *Review of Economic Studies*, Vol. 68, pp. 869–882.

Kox, Henk L.M. (2001), "Sources of Structural Growth in Business Services." CPB Research Memorandum No. 12, The Hague. Available at http://www.cpb.nl/eng/pub/cpbreeksen/memorandum/12/memo12.pdf.

Kox, Henk L.M. and Luis Rubalcaba (2007), "Analysing the Contribution of Business Services to European Economic Growth." MPRA Paper 2003, University Library of Munich, Germany.

Krüger, Jens J (2008), "Productivity and Structural Change: A Review of the Literature," *Journal of Economic Surveys*, Vol. 22, No. 2, pp. 330–363.

Kuznets, Simon (1973), "Modern Economic Growth: Findings and Reflections," *The American Economic Review*, Vol. 63, No. 3, pp. pp. 247–258.

Matsuyama, Kiminori (2013), "Endogenous Ranking and Equilibrium Lorenz Curve Across (ex ante) Identical Countries," *Econometrica*, Vol. 81, No. 5, pp. 2009–2031.

McAllister, Patrick H. and Douglas McManus (1993), "Resolving the Scale Efficiency Puzzle in Banking," *Journal of Banking and Finance*, Vol. 17, No. 2-3, pp. 389–405.

Morikawa, Masayuki (2011), "Economies of Density and Productivity in Service Industries: An Analysis of Personal Service Industries based on Establishment-

Level Data," *The Review of Economics and Statistics*, Vol. 93, No. 1, pp. 179–192.

Moro, Alessio (2012a), "The Structural Transformation between Manufacturing and Services and the Decline in the {US} {GDP} Volatility," *Review of Economic Dynamics*, Vol. 15, No. 3, pp. 402 – 415.

——— (2012b), "Biased Technical Change, Intermediate Goods, and Total Factor Productivity," *Macroeconomic Dynamics*, Vol. 16, pp. 184–203, 4.

Ngai, L. Rachel and Christopher A. Pissarides (2007), "Structural Change in a Multisector Model of Growth," *The American Economic Review*, Vol. 97, No. 1, pp. 429–443.

——— (2008), "Trends in Hours and Economic Growth," *Review of Economic Dynamics*, Vol. 11, No. 2, pp. 239–256.

Ofer, Gur (1973), "Returns to Scale," *Retail Trade*, Vol. 19, No. 4, pp. 363–384.

Raa, Thijs ten and Edward N. Wolff (2000), "Outsourcing of Services and the Productivity Recovery in U.S. Manufacturing in the 1980s and 1990s," *Journal of Productivity Analysis*, Vol. 16, No. 2, pp. 149–165.

Smith, Adam. (Edited by C. J. Bullock) (2001), *Wealth of Nations. Vol. X. The Harvard Classics*: (New York: P.F. Collier and Son, 1909–14).

Srivastava, Aseem (2007), "The Shenzhen Syndrome: Growth Compromises Equity." Available at http://infochangeindia.org/trade-a-development/analysis/the-shenzhen-syndrome-growth-compromises-equity.html.

Triplett, Jack E. and Barry P. Bosworth (2003), "Productivity Measurement Issues in Services Industries: "Baumol's Disease" Has Been Cured," *New York FED Economic Policy Review*, Vol. 9, pp. 23–33.

WDR (2009), *World Development Report 2009: Reshaping Economic Geography*: World Bank.

Wilson, Paul and Kathleen Carey (2004), "Nonparametric Analysis of Returns to Scale in the U.S. Hospital Industry," *Journal of Applied Econometrics*, Vol. 19, No. 4, pp. 505–524.

Zuleta, Hernando and Andrew T. Young (2013), "Labor Shares in a Model of Induced Innovation," *Structural Change and Economic Dynamics*, Vol. 24, No. 0, pp. 112 – 122.

Notes

LEAD FOOTNOTE: Comments from referees vastly improved the paper. It has also benefited from remarks by Poonam Gupta and participants of the 7th Annual Conference on Growth and Development held at the Indian Statistical Institute in December 2011.

[1]In China, considered today as the manufacturing hub of the world economy, the services sector is only a close second to manufacturing.

[2]EU KLEMS reports, for each sub-sector of an economy, price indices, which are used in calculating real sectoral outputs.

[3]Pure business services data in Table 1 include outsourcing activities. Hence some critics point that the growth of business services might just be an 'accounting' phenomenon: the tasks which were performed in-house by the manufacturing firms are now bought from service firms. However, the growth of business services does not seem to be primarily driven by outsourcing. As Kox and Rubalcaba (2007) and Eichengreen and Gupta (2011) note, outsourcing can explain only a small part of the growth of business services. There may be several reasons. First, the IT revolution in the 1970s led to application of technology in novel ways which itself led to creation of *new* services (such as internet, market research and consultancy). Second, as Beyers and Lindahl (1996) have found, the need for specialized knowledge is by far the most important factor behind the demand for producer services. Finally, services rendered by the business services suppliers may be superior to the prior in-house service activities of the outsourcing firm (Kox (2001)). Raa and Wolff (2000) find that the use of business services led to higher total factor productivity growth in manufacturing - clearly indicating the additional benefit of business services over in-house services.

[4]See, for example, Fisher (1939) and Smith (2001).

[5]Furthermore, at the aggregate level, per capita output measured by per capita GDP grows at a constant rate: a Kaldor stylized fact. However, our model does not incorporate physical capital as a factor of production, and hence is silent about other Kaldor facts.

[6]The entry "Services (70-89)" refer to personal services.

[7]Typically, firms with less than 20 employees are taken to be small enterprises.

[8]BLS (2012) finds that in the U.S., workers in manufacturing have median job tenures of about 5-6 years, compared to about 3-5 years in services.

[9]These are the largest economies for which such data was available in the OECD database.

[10]In their two-sector open economy model with a traded sector which is manufacturing and a non-traded sector which is services, Cosar et al. (2010) assume positive turnover costs in manufacturing,

while the services sector is assumed to be frictionless.

[11]Because of the assumed pattern of TFP growth rates, output growth is the least in the services sector.

[12]Another strand of literature uses the framework of non-balanced growth to capture business cycles facts, e.g., (Moro (2012a)) and productivity trends (Moro (2012b) and Duarte and Restuccia (2010)). While these papers focus on effects of non-balanced growth, the current paper deals with sources of non-balanced growth.

[13]Interestingly, for a large public-sector steel conglomerate in India – SAIL (Steel Authority of India Limited), Das and Sengupta (2004) found evidence of negative marginal product of the managerial workforce.

[14]Notice that the marginal product of labor, $\alpha L_{mt}^{\alpha-1} q_{st}^{\beta} - \gamma$ is positive as long as $w_t > 0$.

[15]Substituting the human capital investment function into the household budget constraint, the household's problem can be equivalently cast as: Maximize $\sum_{0}^{\infty} \rho^t \ln[w_t(L_t - L_{t+1}/a_L) + \pi_{mt}]$, subject to $L_t \geq 0$ for $t \geq 1$. For given w_t and π_{mt}, the function $\ln[w_t(L_t - L_{t+1}/a_L) + \pi_{mt}]$ is concave in L_t and L_{t+1}. Hence the overall objective function is concave in $\{L_t\}_1^{\infty}$. The Euler equation is thus a sufficiency condition.

[16]Output growth ranking also holds when manufacturing output is measured in terms of value-added.

[17]The indirect felicity function is $\lambda \ln \lambda + (1-\lambda)\ln(1-\lambda) - (1-\lambda)\ln p_{st}^h + \ln E_t$, where E_t is the household expenditure. Using the budget constraint, it is equal to: $\lambda \ln \lambda + (1-\lambda)\ln(1-\lambda) - (1-\lambda)\ln p_{st}^h + \ln\left[w_t\left(L_t - \dfrac{L_{t+1}}{a_L}\right) + \pi_{mt}\right]$, which is concave in L_t and L_{t+1}. Hence, the sufficiency condition is met. The same transversality condition holds.

[18]In the light of (22), $g_{\bar{L}_t} = \rho a_L \dfrac{1+\bar{\Phi}(\bar{L}_t)}{1+\bar{\Phi}(\bar{L}_{t+1})} < \rho a_L$. Hence the dynamics of H_t is qualitatively same as in the base model. The transversality condition holds along the saddle path.

[19]It amounts to hypothesizing that if more disaggregated data on hybrid services were available, it would exhibit higher growth rate of the business-services segment, vis-a-vis consumption services.

[20]The inequality (26) is not restrictive as long as the elasticity of substitution among business services is sufficiently large – which is eminently plausible.

[21]It is worth noting that the preceding analysis does not consider the dynamic effects of the use of manufactures in services production such as an embodied technological progress; otherwise, it would have tended to enhance the growth rate of the services sector.

[22]That is, the so-called Baumol's disease (see Baumol (1967)) has either been "cured" or not struck.

[23] In terms of (A.1), both $\Phi(\bar{L}_t)$ and $\Phi(\bar{L}_{t+1})$ approach 0 or $(\alpha/\beta)(1-\alpha-\beta)$ as $\bar{L}_t \to 0$ or ∞.

[24] The $\Psi(\cdot)$ curve may have more complex curvature, but, the important feature is that it lies below $H_t = \rho$ line.

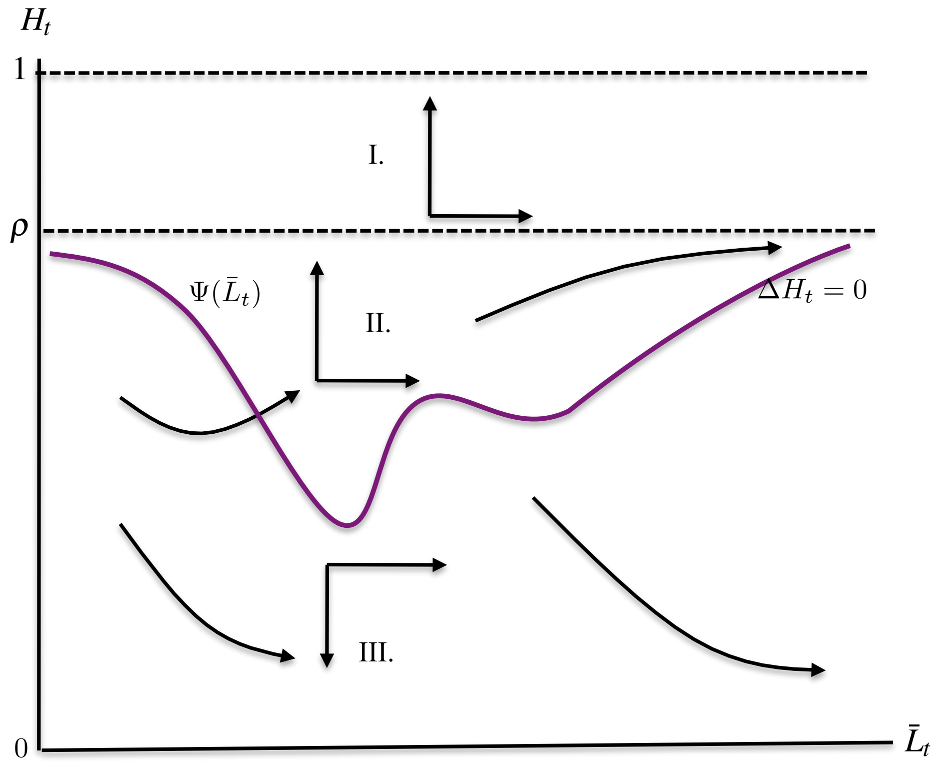

Figure 1: Dynamics of H_t